Similarity Renormalization Group Developments

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In collaboration with R. J. Furnstahl and R. J. Perry

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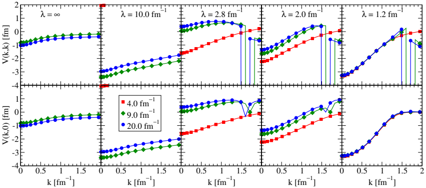
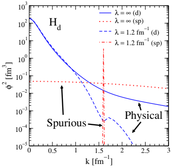
Topics:

- Universal SRG Potentials beyond A=2
- Local projections of SRG potentials.
- Progress towards controlling the evolution of initial 3NF.

Decoupling of Spurious Deep Bound States with SRG

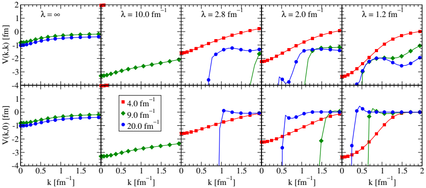
K. A. Wendt, R. J. Furnstahl, R. J. Perry, PRC 83, 034005 (2011)

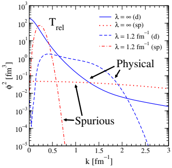




$$\frac{\partial}{\partial s} H = [[H_{Diag}, H], H]$$

- H_{Diag} :"Wegner", Diagonal of H
- H_{Diag} produces desired decoupling and a universal low momentum form.

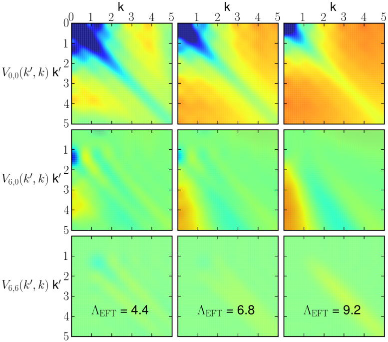




$$\frac{\partial}{\partial s} H = [[T_{rel}, H], H]$$

- T_{rel} :Relative Kinetic Energy
- T_{rel} does not!
- This is a symptom of T_{rel} not renormalizing past a bound state scale!

2 + 3 Body Universal Interactions from SRG
A 1D model



- 2 momentum dof's.
- Express in polar coordinates.
- Expand angle in a Fourier series.
- 1D analog to a hyper-radial basis

Visualizing 3-Body Universality

- Model
  - 2-Body Only. Attractive long and a repulsive short range Gaussian.
  - Replace short range with a EFT, fit 2 and 3 body data to N^2LO for 3 different regulators.
- Each initial interaction is different
- But they produce equivalent observables

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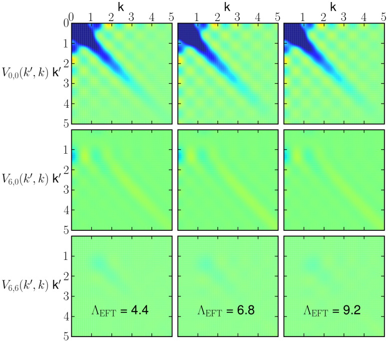

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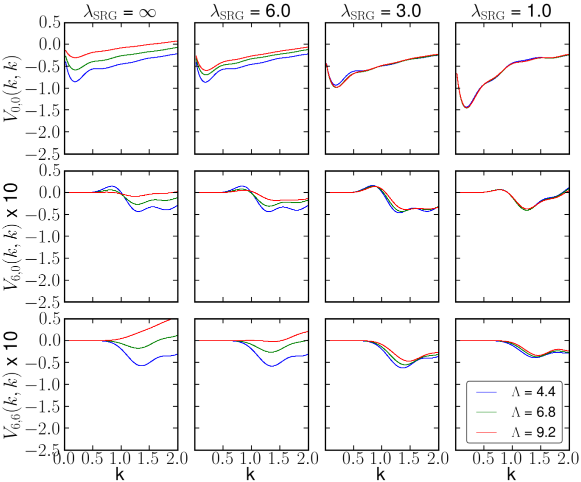

2- and 3-body components are included in figure.

- Initial interactions are very different
- Final interactions have same low momentum form
- This may be exploitable to understand and control the growth of many-body forces from SRG!

SRG Effects at Long Range: Local Projections

- How does SRG effect interactions at long range?
  - Interactions are non-local $\rightarrow$ hard to visualize what happens at long range (or short range) $\rightarrow$ hard to develop intuition.

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$$\begin{aligned} \overline{O}(\mathbf{r}) &= \int d^3\mathbf{r}'\, O(\mathbf{r},\mathbf{r}') \ \text{(Suggested by Andreas Nogga)} \\ \widetilde{O}(\mathbf{r}) &= \int d^3\mathbf{r}'\, O(\mathbf{r}+\frac{\mathbf{r}'}{2},\mathbf{r}-\frac{\mathbf{r}'}{2}) \end{aligned}$$

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  - For a local operator, $O(\mathbf{r},\mathbf{r}') = O(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}')$, these projections are identity:

$$\begin{aligned}\overline{O}(\mathbf{r}) &= \int d^3\mathbf{r}'\, O(\mathbf{r})\delta(\mathbf{r}-\mathbf{r}') = O(\mathbf{r}) \\ \widetilde{O}(\mathbf{r}) &= \int d^3\mathbf{r}'\, O(\mathbf{r}+\frac{\mathbf{r}'}{2})\delta(\mathbf{r}') = O(\mathbf{r})\end{aligned}$$

  - Can look at things such as $\overline{V}_\lambda$ or $\overline{V}_\lambda - \overline{V}_{\lambda=\infty}$

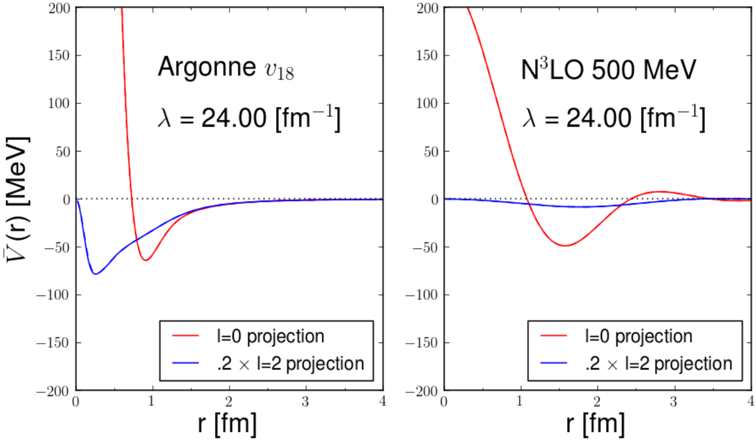
SRG Effects at Long Range: Local Projections

- How we get to these projections from our current evolutions:

$$
\begin{aligned}
\overline{O}(\mathbf{r}) &= \int d^3\mathbf{r}'\, O_\lambda(\mathbf{r},\mathbf{r}') \\
&\propto \sum_{l,m} Y_l^m(\Omega_{\mathbf{r}}) \int k^2 dk\; j_l(kr) O_{l,0}(k,0)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{O}(\mathbf{r}) &= \int d^3\mathbf{r}'\, O_\lambda(\mathbf{r}+\frac{\mathbf{r}'}{2},\mathbf{r}-\frac{\mathbf{r}'}{2}) \\
&\propto \sum_{l,m} Y_l^m(\Omega_{\mathbf{r}}) \sum_{\substack{l_1,m_1\\ l_2,m_2}} (-1)^{l_2+m_2} \int k^2 dk\; j_l(2kp) O_{l_1,l_2}(k,k) \times \\
&\sqrt{\frac{(2l+1)(2l_1+1)(2l_2+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_1 & l_2 & l \\ m_1 & m_2 & m \end{pmatrix}
\end{aligned}
$$

SRG Effects at Long Range: Local Projections



Manipulation of the Evolution of the Three Body Force

- Roth et al. found that initial three-body forces can induce significant many-body forces after SRG evolution.
  - Verified by Jurgenson et al.
- Want to explore methods to control such forces.

$$\frac{d}{ds}H = [[T_{rel}, H], H] \quad (\lambda = s^{-4})$$

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$$\frac{d}{ds}H = [[T_{rel} + \color{red}{O}, H], H] \quad (\lambda = s^{-4})$$

- Roth tried $\frac{d}{ds}H = [[T_{rel}, V_s^{(2)}], H_s]$, did not fix the problem.
- Various possibilities: H_{Diag}^{ms}, $V_\lambda^{(3)}$,$V_{\lambda=\infty}^{(3)}$, combinations of these with different regulators
- A playground in 1D
  - $V^{(2)}$ is the sum of an attractive long- and repulsive short-range Gaussian
  - $V^{(3)}$ is a long range ($> V^{(2)}$) Gaussian in the radius of the Jacobi coordinates.

Manipulation of the Evolution of the Three Body Force

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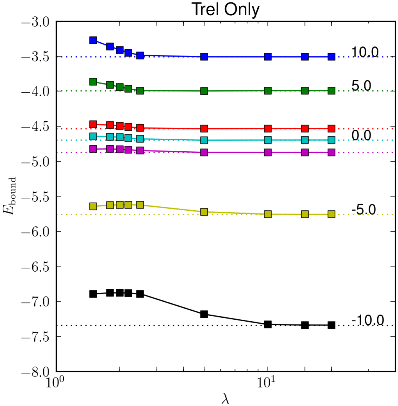

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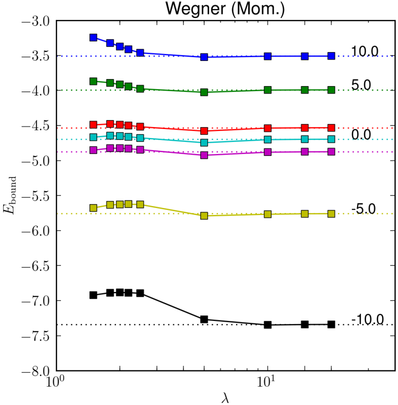
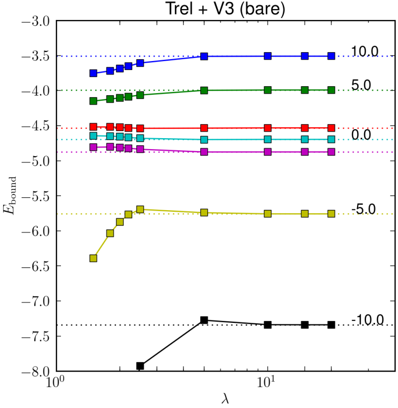
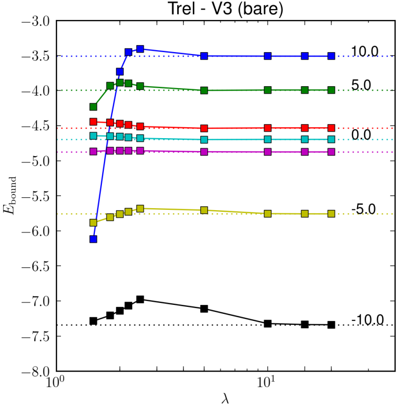


Manipulation of the Evolution of the Three Body Force

$$\frac{d}{ds}H = [[T_{rel} + V^{(3)}_{s=0}, H], H]$$

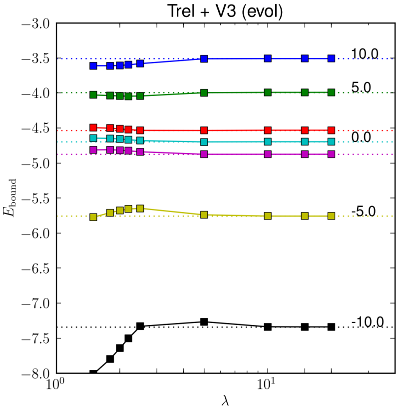
$$\frac{d}{ds}H = [[T_{rel} - V^{(3)}_{s=0}, H], H]$$



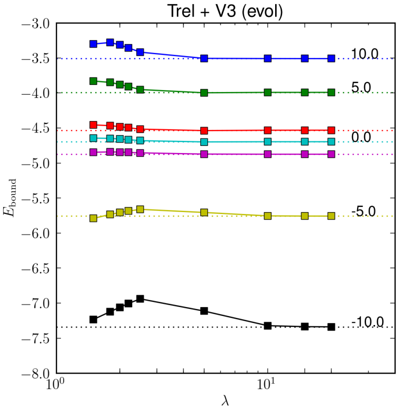


Manipulation of the Evolution of the Three Body Force

$$\frac{d}{ds}H = [[T_{rel} + V_s^{(3)}, H], H]$$

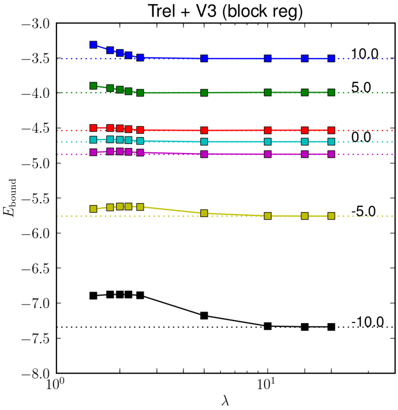


$$\frac{d}{ds}H = [[T_{rel} - V_s^{(3)}, H], H]$$

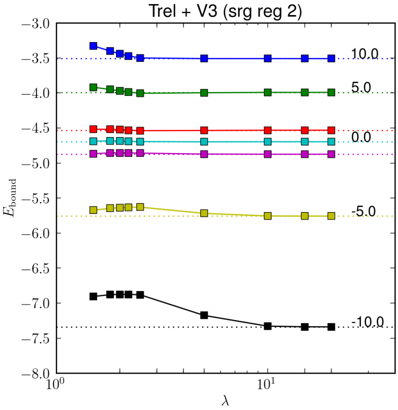


Manipulation of the Evolution of the Three Body Force

$$\frac{d}{ds}H = [[T_{rel} + V_{s=0}^{(3)(\text{block reg})}, H], H]$$



$$\frac{d}{ds}H = [[T_{rel} + V_{s=0}^{(3)(\text{SRG reg})}, H], H]$$



Things in progress

- SRG connections between chiral and pionless EFT.
- Hyper-radial momentum space evolutions for A>3.
  - Test of low momentum universality in 3D for A-body forces.
  - Visualization of SRG evolution beyond A>3.
- Extend local projection analysis to A>2.
  - Find and use other local “projections”.
- Continue work on controlling induced 4-body forces (from initial 3-body)
- QMC methods for chiral/srg interactions
  - Ψ is simple after SRG (correlations suppressed)
  - Non-local makes other things harder (lots of spectator and conservation delta functions to handle, etc)
  - VMC Soon, LR-DMC later.